FPT in Economics

The Stable Marriage Problem

Meirav Zehavi

Outline

- Introduction
- Basics
- Satisfaction Optimization
- Size Optimization
- Minimum BPs, Manipulation
- Roommate, Hospitals/Residents

Introduction

Input. A set of men *M*, and a set of women *W*.

Introduction

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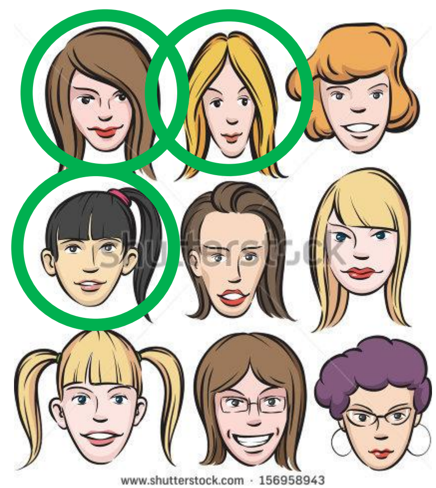
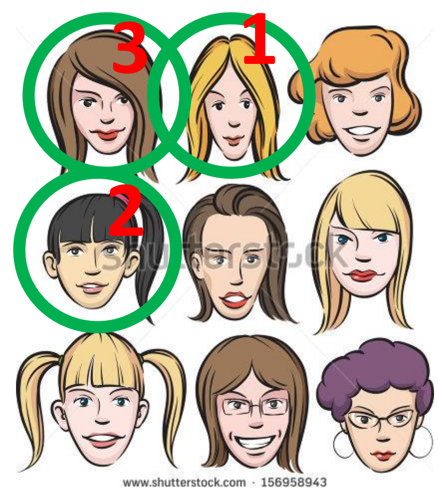


Introduction

Input. A set of men M, and a set of women W.
Every agent has a set of acceptable partners.
The acceptable partners are ranked.
Bijective function $p_m : W' \rightarrow \{1,\ldots,|W'|\}$.



Introduction

Input.

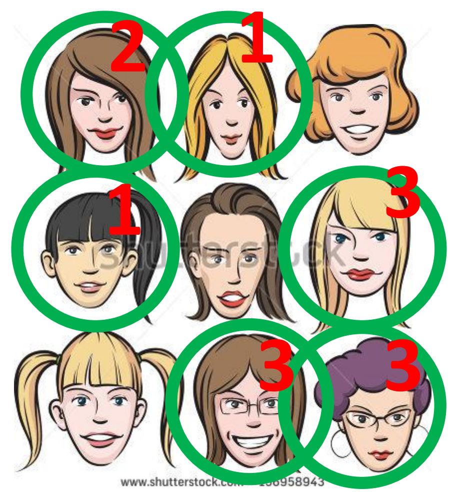
- Complete/**incomplete** lists.

Introduction

Input.

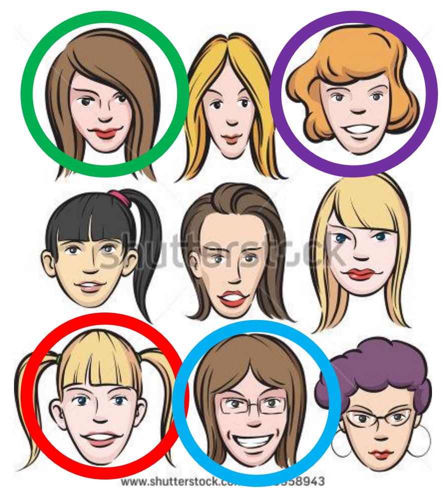
- Complete/**incomplete** lists.
- Ties allowed/**forbidden**.

Surjective function $p_m : W' \rightarrow \{1,\ldots,t\}$, $t \leq |W'|$.



Introduction

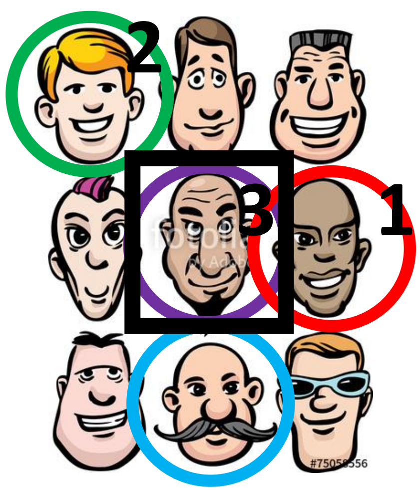
Matching. A set of pairwise-disjoint pairs, each consisting of a man and a woman that find each other acceptable.

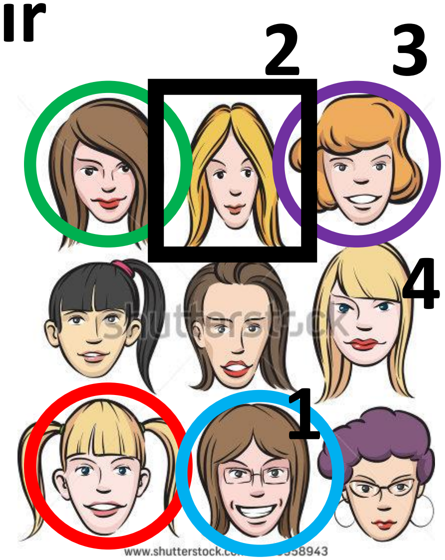


Introduction

Blocking pair. A pair (m,w) blocks a matching if m and w prefer being matched to each other to their current ``status''.

blocking pair





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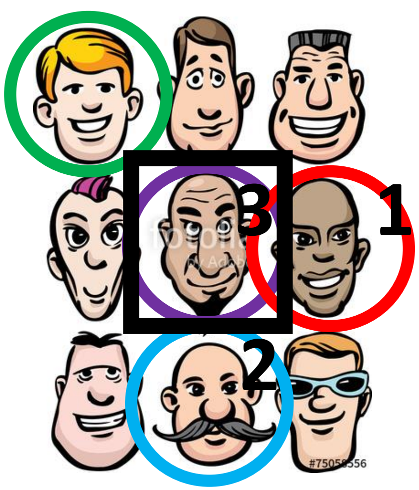
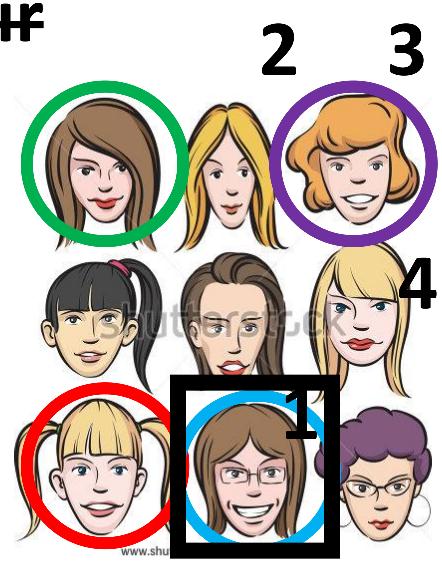
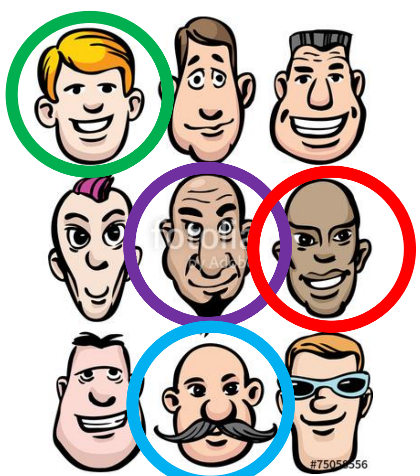
~~**blocking pair**~~

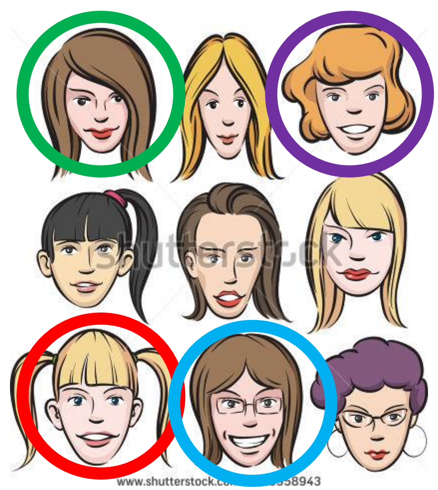




Introduction

Stable matching. A matching that has no blocking pair.





Introduction

Stable Marriage problem. Find a stable matching.

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Stable Marriage problem. Find a stable matching.

Nobel Prize in Economics, 2012. Awarded to Shapley and Roth ``for the theory of stable allocations and the practice of market design.''

Introduction

Applications.

- Matching hospitals to residents.
- Matching students to colleges.
- Matching kidney patients to donors.
- Matching users to servers in a distributed Internet service.

Introduction

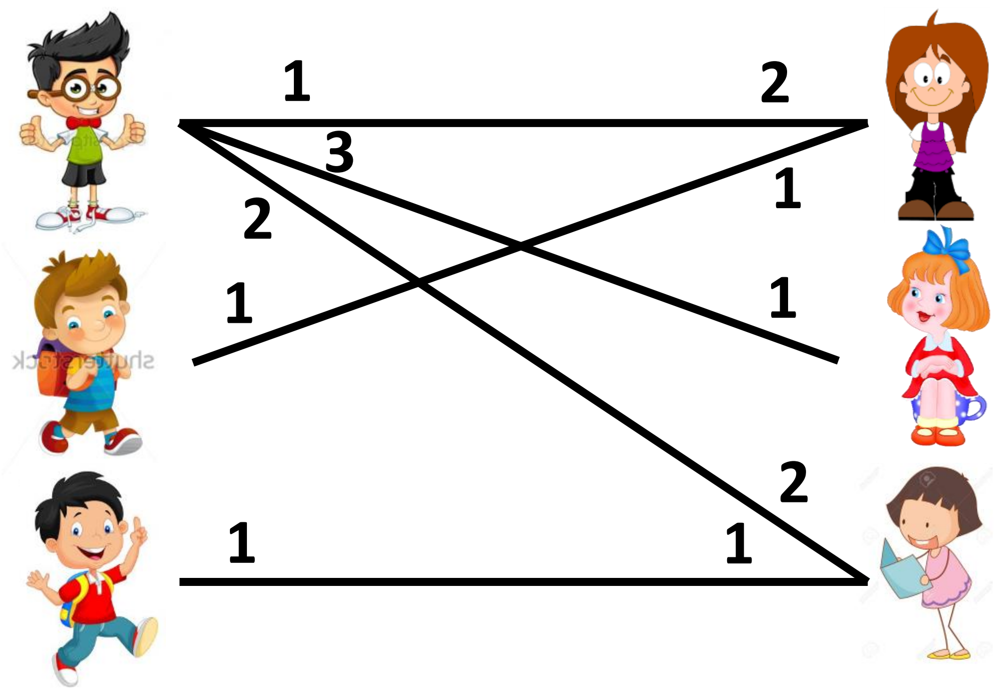
Books.

- Gusfield and Irving, *The stable marriage problem–structure and algorithms*, 1989.
- Knuth, *Stable marriage and its relation to other combinatorial problems*, 1997.
- Manlove, *Algorithmics of matching under preferences*, 2012.

Surveys. Iwama and Miyazaki, 2008; Gupta, Roy, Saurabh and Zehavi, 2017.

Introduction

Primal graph. A bipartite graph with bipartition (M,W), where m and w are adjacent iff they find each other acceptable.



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Basics

Proposition. A stable matching can be found in time $O(n^2)$. [Gale and Shapley, 1962]

→ A stable matching always exists.

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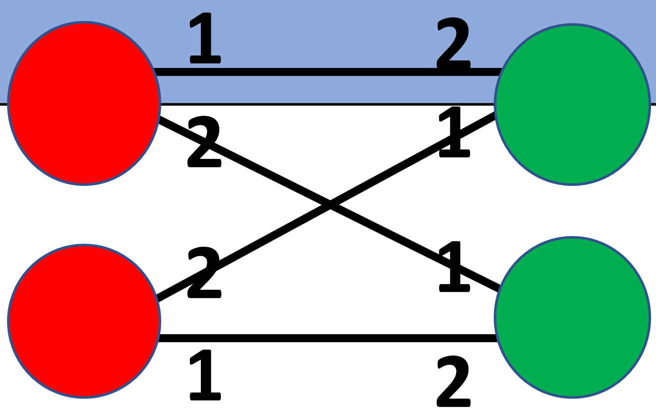

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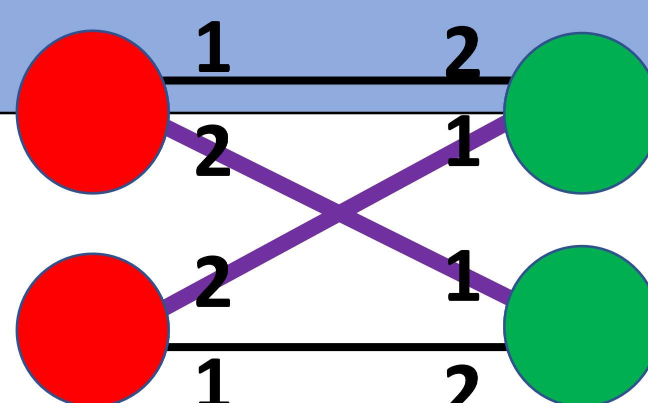

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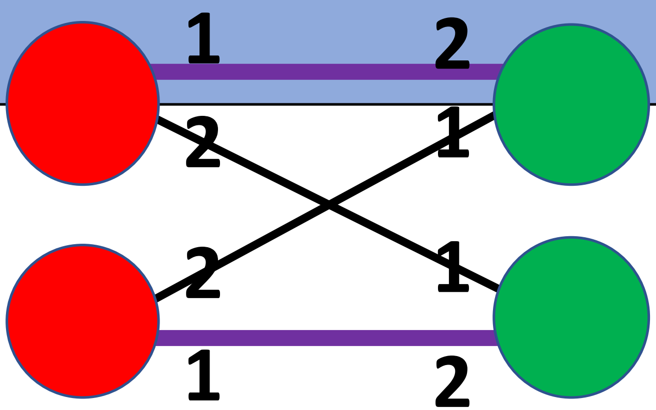

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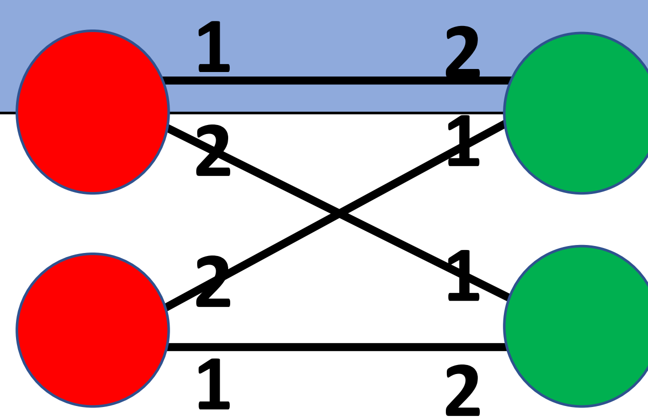
duplicate



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→ A stable matching always exists.

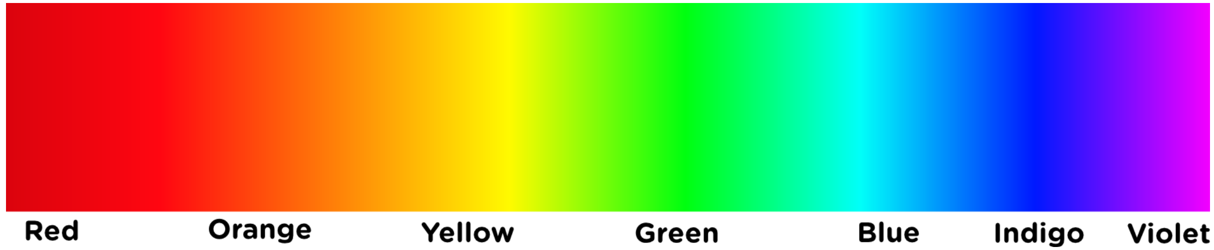
There can be an exponential number of stable matchings. [Gusfield and Irving, 1989]

Proposition. All stable matchings match the same set of agents. [Gale and Sotomayor, 1985]

Basics

A ``spectrum'' of stable matchings, where the two extremes are the **man-optimal stable matching** and the **woman-optimal stable matching**.

man-optimal **woman-optimal**



Basics

A ``spectrum'' of stable matchings, where the two extremes are the **man-optimal stable matching** and the **woman-optimal stable matching**.

Man-optimal stable matching μ_M. For every stable matching μ and man m, either m is unmatched by both μ_M and μ, or $p_m(\mu_M(m)) \leq p_m(\mu(m))$.

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→ Unique.

Basics

A ``spectrum'' of stable matchings, where the two extremes are the **man-optimal stable matching** and the **woman-optimal stable matching**.

Man-optimal stable matching μ_M. For every stable matching μ and man m, either m is unmatched by both μ_M and μ, or $p_m(\mu_M(m)) \leq p_m(\mu(m))$.

Proposition. μ_M and μ_W exist, and can be found in time $O(n^2)$. [Gale and Shapley, 1962]

Basics

Rotation. A μ-rotation is an ordered sequence $\rho = ((m_0,w_0),(m_1,w_1),\ldots,(m_{r\text{-}1},w_{r\text{-}1}))$ such that for all i,

- $(m_i,w_i) \in \mu$, and
- $w_{(i+1)\text{mod } r}$ is the woman succeeding w_i in m_i's preference list who prefers being matched to m_i to her current status.

m_i: … w_i … $w_{(i+1)\text{mod } r}$ …

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ρ is a **rotation** if it is a μ-rotation for some μ.

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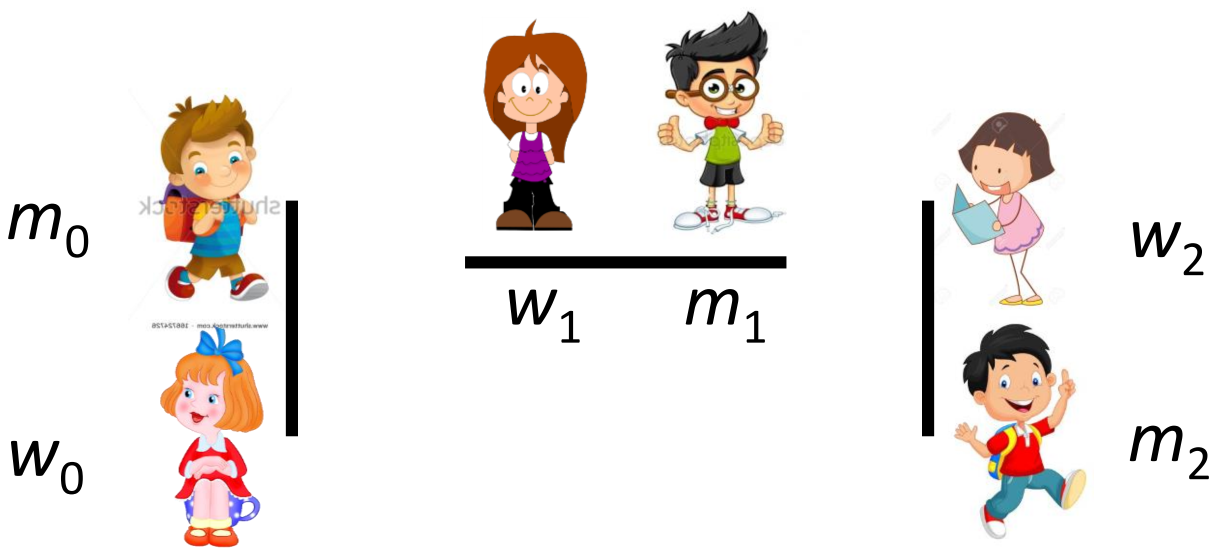
ρ is a **rotation** if it is a μ-rotation for some μ.

The set of all rotations is denote by R.
It is known that $|R| \le n^2$.

Basics

Rotation elimination. Consider a μ-rotation ρ = $((m_0,w_0),(m_1,w_1),\ldots,(m_{r\text{-}1},w_{r\text{-}1}))$.
The elimination of is the operation that modifies μ by matching each m_i with $w_{(i+1)\mathrm{mod}\ r}$ rather than w_i.



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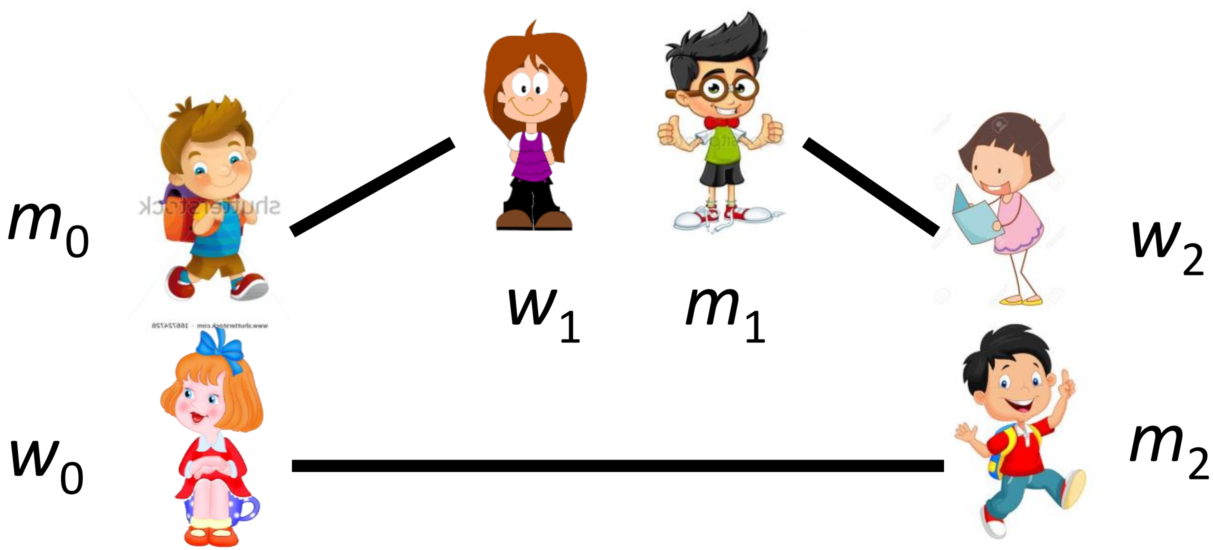

Basics

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The elimination of is the operation that modifies μ by matching each m_i with $w_{(i+1) \bmod r}$ rather than w_i.

Rotation elimination results in a stable matching. [Irving and Leather, 1986]

Basics

Proposition. Let μ be a stable matching. There is a unique subset of R, denoted by $R(\mu)$, such that starting from μ_M, there is an order in which the rotations in $R(\mu)$ can be eliminated to obtain μ. [Irving and Leather, 1986]

Basics

Rotation poset. $\prod=(R,\prec)$, where $\prec$ is a partial order on R such that $\rho \prec \rho'$ iff for every stable matching μ, if ρ' is in $R(\mu)$, then ρ is also in $R(\mu)$.

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- **Elimination compatible with** $\prec$.

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Rotation poset. $\prod=(R,\prec)$, where $\prec$ is a partial order on R such that $\rho \prec \rho'$ iff for every stable matching μ, if ρ' is in $R(\mu)$, then ρ is also in $R(\mu)$.

- **Elimination compatible with $\prec$.**
- **Closed set *R'*.** If $\rho \in R'$, then $\rho' \in R'$ for all $\rho' \prec \rho$.

Basics

Proposition. Let R' be a closed set. Starting with μ_M, eliminating the rotations in R' in any $\prec$-compatible order is valid—at each step, where the current stable matching is μ, the rotation we eliminate next is a μ-rotation.

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Proposition. Let R' be a closed set. Starting with μ_M, eliminating the rotations in R' in any $\prec$-compatible order is valid—at each step, where the current stable matching is μ, the rotation we eliminate next is a μ-rotation.

Moreover, all $\prec$-compatible orders in which one eliminates the rotations in R' result in the same stable matching. [Irving and Leather, 1986]

Basics

Rotation digraph. A compact representation of Π. The rotation digraph is the DAG of minimum size whose transitive closure is isomorphic to Π.

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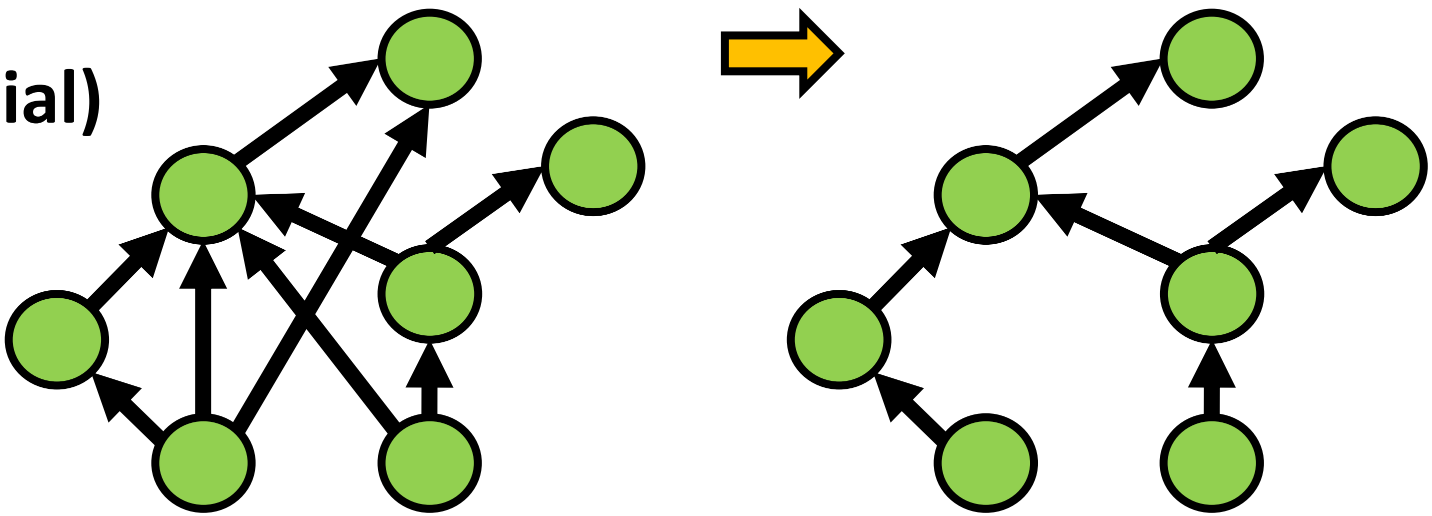
Π
(partial)



Basics

Rotation digraph. A compact representation of Π. The rotation digraph is the DAG of minimum size whose transitive closure is isomorphic to Π.

Proposition. The rotation digraph can be computed in time $O(n^2)$. [Irving, Leather and Gusfield, 1987]

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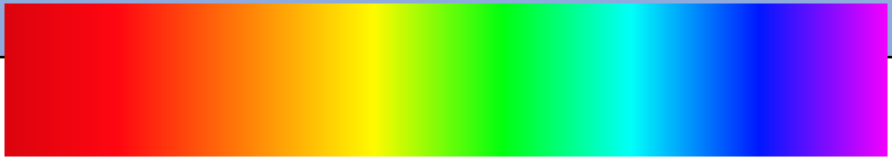
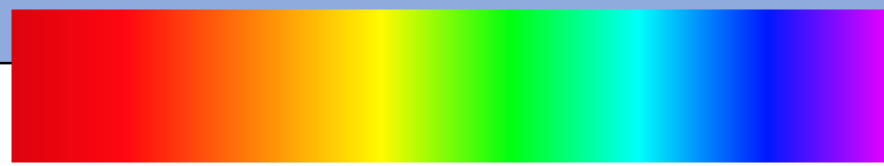
Satisfaction Optimization

Recall. man-optimal woman-optimal



Satisfaction Optimization

Recall. man-optimal woman-optimal

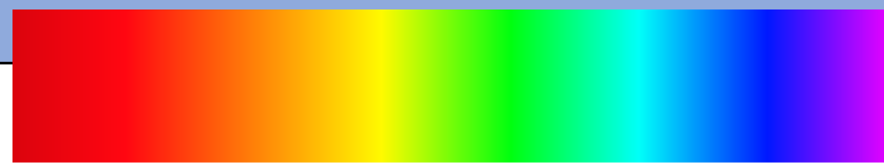


Three satisfaction optimization approaches.

- Globally desirable.
- Fair towards both sides.
- Desirable by both sides.

Satisfaction Optimization

Recall. man-optimal woman-optimal



Three satisfaction optimization approaches.

- Globally desirable.
- Fair towards both sides.
- Desirable by both sides.

No ties.

Satisfaction Optimization

Egalitarian stable matching. Minimize

$$\mathrm{e}(\mu) = \sum_{(m,w)\in\mu} \left(pm(w) + pw(m)\right).$$

Satisfaction Optimization

Egalitarian stable matching. Minimize

$$e(\mu) = \sum_{(m,w)\in\mu} \left(pm(w) + pw(m)\right).$$

Comment. In the presence of ties, can use either the definition above or one where each unmatched agent *a* contributes $|\mathrm{domain}(p_a)|+1$ to the sum.

Satisfaction Optimization

Proposition. Egalitarian Stable Marriage is solvable in polynomial time. [Irving, Leather and Gusfield, 1987]

Satisfaction Optimization

Proposition. Egalitarian Stable Marriage is solvable in polynomial time. [Irving, Leather and Gusfield, 1987]

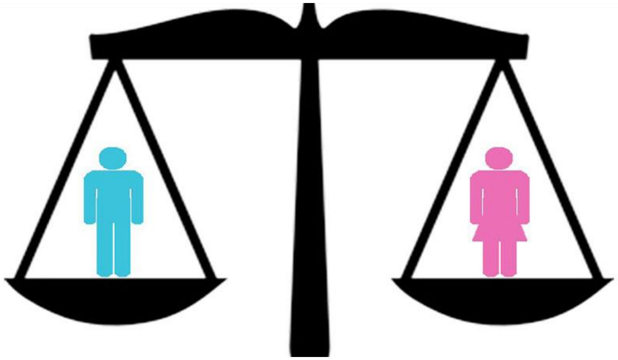
In the presence of ties (NP-hard):

Proposition. Egalitarian Stable Marriage is FPT parameterized by ``total length of ties''. [Marx and Schlotter, 2010]

Satisfaction Optimization

Sex-equality measure.

$$\delta(\mu) = \sum_{(m,w)\in\mu} p_m(w) - \sum_{(m,w)\in\mu} p_w(m).$$



Satisfaction Optimization

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Sex-Equal Stable Marriage. Find a stable matching that attains $\Delta = \min_\mu |\delta(\mu)|$.

Satisfaction Optimization

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Sex-Equal Stable Marriage. Find a stable matching that attains $\Delta = \min_\mu |\delta(\mu)|$.

Proposition. Sex-Equal Stable Marriage is NP-hard. [Kato, 1993]

Satisfaction Optimization

Propositions. [Gupta, Saurabh and Zehavi, 2017]

1. Sex-Equal Stable Marriage is W[1]-hard w.r.t *tw*, the treewidth of the primal graph. Moreover, unless ETH fails, it cannot be solved in time $f(tw)n^{o(tw)}$.

Satisfaction Optimization

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1. Sex-Equal Stable Marriage is W[1]-hard w.r.t *tw*, the treewidth of the primal graph. Moreover, unless ETH fails, it cannot be solved in time $f(tw)n^{o(tw)}$.
2. Sex-Equal Stable Marriage can be solved in time $n^{O(tw)}$.

Satisfaction Optimization

Propositions. [Gupta, Saurabh and Zehavi, 2017]

1. Sex-Equal Stable Marriage is solvable in time $2^{tw} n^{O(1)}$, where tw is the treewidth of the rotation digraph.

Satisfaction Optimization

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1. Sex-Equal Stable Marriage is solvable in time $2^{tw}n^{O(1)}$, where tw is the treewidth of the rotation digraph.
2. Unless SETH fails, Sex-Equal Stable Marriage cannot be solved in time $(2-\varepsilon)^{tw}n^{O(1)}$.

Satisfaction Optimization

Proposition. Sex-Equal Stable Marriage is solvable in time $(2^{\alpha n}+2^{\beta})^{tw}n^{O(1)}$ for $\alpha=(5-\sqrt{24})(t-2+\varepsilon)$ and $\beta=(t-1)/2\varepsilon$, where t is the maximum length of a list.
[McDermid and Irving, 2014]

Satisfaction Optimization

Balance measure. $bal(\mu) =$

$$\max\{\sum_{(m,w)\in\mu} p_m(w), \sum_{(m,w)\in\mu} p_w(m)\}.$$

Satisfaction Optimization

Balance measure. $bal(\mu) =$

$$\max\{ \sum_{(m,w)\in\mu} p_m(w) , \sum_{(m,w)\in\mu} p_w(m)\} .$$

Balanced Stable Marriage. Find a stable matching that attains Bal=$\min_\mu bal(\mu)$.

Satisfaction Optimization

Balance measure. $bal(\mu)$ =

$$\max\{\sum_{(m,w)\in\mu} p_m(w)\,,\ \sum_{(m,w)\in\mu} p_w(m)\}\,.$$

Balanced Stable Marriage. Find a stable matching that attains Bal=$\min_\mu bal(\mu)$.

Proposition. Balanced Stable Marriage is NP-hard. [Feder, 1990]

Satisfaction Optimization

Construction of a family of instances where no stable matching is both sex-equal and balanced. [Manlove, 2013]

Satisfaction Optimization

Propositions. [Gupta, Saurabh and Zehavi, 2017]

1. Balanced Stable Marriage is W[1]-hard w.r.t *tw*, the treewidth of the primal graph. Moreover, unless ETH fails, it cannot be solved in time $f(tw)n^{o(tw)}$.
2. Balanced Stable Marriage can be solved in time $n^{O(tw)}$.

Satisfaction Optimization

Propositions. [Gupta, Saurabh and Zehavi, 2017]

1. Balanced Stable Marriage is solvable in time $2^{tw}n^{O(1)}$, where *tw* is the treewidth of the rotation digraph.
2. Unless SETH fails, Balanced Stable Marriage cannot be solved in time $(2-\varepsilon)^{tw}n^{O(1)}$.

Satisfaction Optimization

$$O_M = \sum_{(m,w)\in\mu_M} p_m(w);\ \ O_W = \sum_{(m,w)\in\mu_W} p_w(m).$$

Parameters.

- $t_1 = k - \min\{O_M, O_W\}$.
- $t_2 = k - \max\{O_M, O_W\}$.

Satisfaction Optimization

Propositions. [Gupta, Roy, Saurabh and Zehavi, 2017]

1. Balanced Stable Marriage admits a kernel with at most $3t_1$ men, $3t_1$ women, and such that each agent has at most $2t_1$+1 acceptable partners.
2. Balanced Stable Marriage is solvable in time $8^{t_1}n^{O(1)}$.
3. Balanced Stable Marriage is W[1]-hard w.r.t. t_2.

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Size Optimization

SMTI. Stable Marriage with <u>Ties</u> and Incomplete lists.

max-SMTI. Find a stable matching of max size.
min-SMTI. Find a stable matching of min size.

Size Optimization

SMTI. Stable Marriage with Ties and Incomplete lists.

max-SMTI. Find a stable matching of max size.
min-SMTI. Find a stable matching of min size.

Proposition. max-SMTI and min-SMTI are NP-hard. [Irving, Iwama, Manlove, Miyazaki and Moitra, 2002]

Size Optimization

Propositions. [Marx and Schlotter, 2010]

1. max-SMTI is FPT w.r.t. total length of ties.
2. max-SMTI is W[1]-hard w.r.t. number of ties.

In addition, they studied strict and permissive local search versions of max-SMTI.

Size Optimization

Propositions. [Gupta, Saurabh and Zehavi, 2017]

1. max-SMTI and min-SMTI are W[1]-hard w.r.t *tw*, the treewidth of the primal graph. Moreover, unless ETH fails, they cannot be solved in time $f(tw)n^{o(tw)}$.
2. max-SMTI and min-SMTI can be solved in time $n^{O(tw)}$.

Size Optimization

Proposition. max-SMTI and min-SMTI admit a kernel of size $O(k^2)$, where k is solution size.
[Adil, Gupta, Roy, Saurabh and Zehavi, 2017]

Size Optimization

Proposition. max-SMTI and min-SMTI admit a kernel of size $O(k^2)$, where k is solution size.
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Proposition. max-SMTI is FPT parameterized by number of ``agent types''.
[Meeks and Rastegari, 2017]

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Minimum BPs, Manipulation

Stable Marriage with Covering Constraints (SMC). Given an instance of Stable Marriage with subsets $M^* \subseteq M$ and $W^* \subseteq W$, find a matching with minimum number of blocking pairs where $M^* \cup W^*$ are matched.

Minimum BPs, Manipulation

Mnich and Schlotter, 2017. Parameters:

- b – # blocking pairs.
- $|M^*|$, $|W^*|$.
- Δ_M (Δ_W) – max length of lists of men (women).

Minimum BPs, Manipulation

Mnich and Schlotter, 2017. Parameters:

- b – # blocking pairs.
- $|M^*|$, $|W^*|$.
- Δ_M (Δ_W) – max length of lists of men (women).

Proposition. SMC is W[1]-hard w.r.t. $b+|W^*|$ even if $|M^*|=0$ and $\Delta_M=\Delta_W=3$.

Proposition. SMC is FPT w.r.t. b if $\Delta_W=2$. Moreover, SMC is FPT w.r.t. $|M^*|+|W^*|$ if $\Delta_W=2$.

Minimum BPs, Manipulation

Manipulation. Stable Extension of Partial Matching (SEOPM).

In. Instance of Stable Marriage, partial matching μ.

Q. Does there exist a set of lists for women, so that when this set is used, Gale-Shapley algorithm returns a matching that extends μ?

Minimum BPs, Manipulation

Manipulation. Stable Extension of Partial Matching (SEOPM).
In. Instance of Stable Marriage, partial matching μ.
Q. Does there exist a set of lists for women, so that when this set is used, Gale-Shapley algorithm returns a matching that extends μ?

Proposition. SEOPM is NP-hard. [Kobayashi and Matsui, 2010]

Minimum BPs, Manipulation

Manipulation. Stable Extension of Partial Matching (SEOPM).

In. Instance of Stable Marriage, partial matching μ.

Q. Does there exist a set of lists for women, so that when this set is used, Gale-Shapley algorithm returns a matching that extends μ?

Proposition. SEOPM is solvable in time $2^{O(n)}$. [Gupta and Roy, 2016]

Minimum BPs, Manipulation

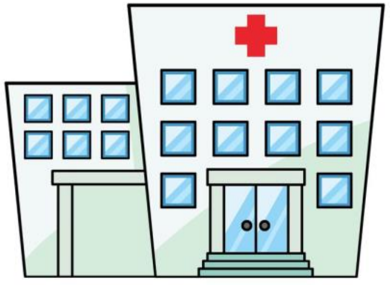
max-Size min-BP SMI. Given an instance of Stable Marriage, find a maximum matching with minimum number of BPs among all maximum matchings.

Minimum BPs, Manipulation

max-Size min-BP SMI. Given an instance of Stable Marriage, find a maximum matching with minimum number of BPs among all maximum matchings.

Proposition. max-Size min-BP SMI is FPT parameterized by number of ``agent types''.
[Meeks and Rastegari, 2017]

Outline



- Introduction
- Basics
- Satisfaction Optimization
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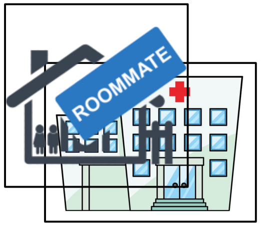
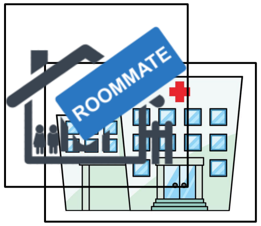
- Roommate, Hospitals/Residents



Roommate, Hospitals/Res.

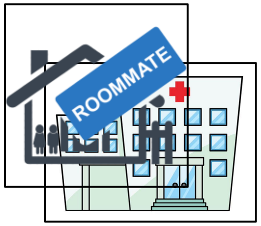
Stable Roommate. Given a set of agents, each ranking a subset of other agents, **determine** if there exists a stable matching.



Roommate, Hospitals/Res.

Stable Roommate. Given a set of agents, each ranking a subset of other agents, **determine** if there exists a stable matching.

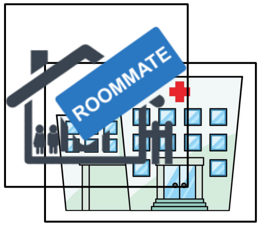
- Without ties, polynomial time. [Irving, 1985]
- With ties, NP-hard. [Ronn, 1990]



Roommate, Hospitals/Res.

Proposition. Egalitarian Stable Roommate with Ties is FPT parameterized by *k*-*n*, where *k* is the solution value. [Chen, Hermelin, Sorge and Yedidson, 2017]

(Unmatched agents contribute, else para-NP-hard.)

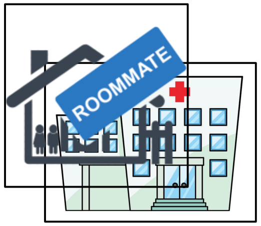


Roommate, Hospitals/Res.

Proposition. Egalitarian Stable Roommate with Ties is FPT parameterized by k-n, where k is the solution value. [Chen, Hermelin, Sorge and Yedidson, 2017]

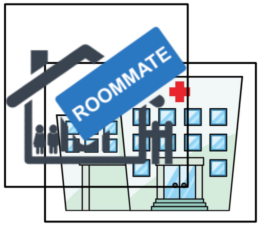
(Unmatched agents contribute, else para-NP-hard.)

Proposition. min-BP Stable Roommate is W[1]-hard w.r.t. b, the number of blocking pairs. Moreover, unless ETH fails, it cannot be solved in time $f(b)n^{o(b)}$. [Chen, Hermelin, Sorge and Yedidson, 2017]



Roommate, Hospitals/Res.

Proposition. max-SRTI and min-SRTI admit a kernel of size $O(k^2)$, where k is the size of a maximum matching. [Adil, Gupta, Roy, Saurabh and Zehavi, ‘17]



Roommate, Hospitals/Res.

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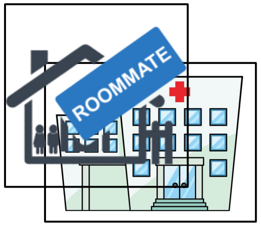
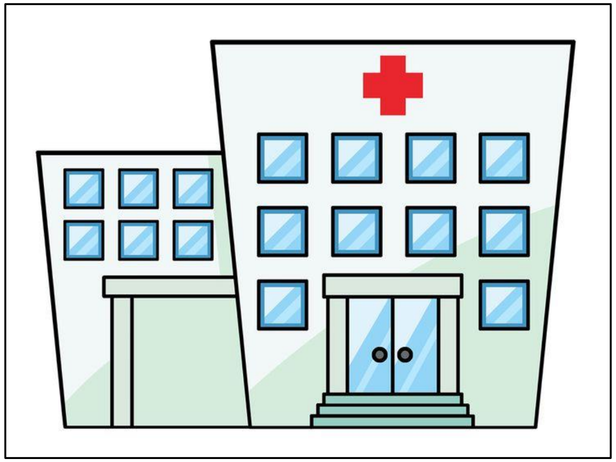
Proposition. max-SRTI is FPT w.r.t. number of ``agent types’’. [Meeks and Rastegari, 2017]

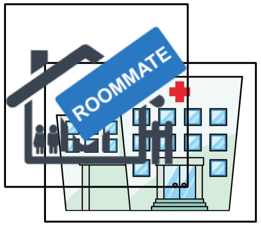


Roommate, Hospitals/Res.

Hospitals/Residents. A set of hospitals *H*, and a set of residents *R*. Every agent has a ranked set of acceptable partners. Every hospital has a lower quota and an upper quota.

Matching. Every resident is matched at most once, and every hospital is matched according to its specification.

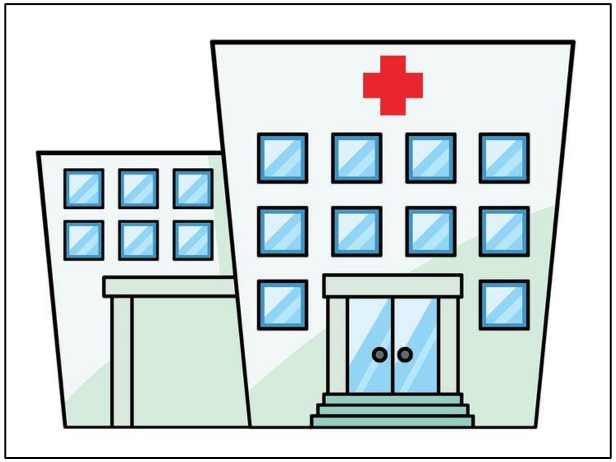


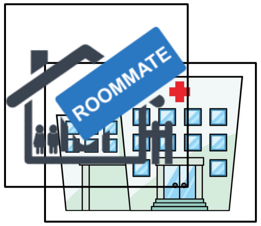


Roommate, Hospitals/Res.

Hospitals/Residents. A set of hospitals *H*, and a set of residents *R*. Every agent has a ranked set of acceptable partners. Every hospital has a lower quota and an upper quota.

Blocking pair (*h*,*r*). *h* either has free space or prefers *r* to an assigned resident; *r* prefers being assigned to *h* to the current status.

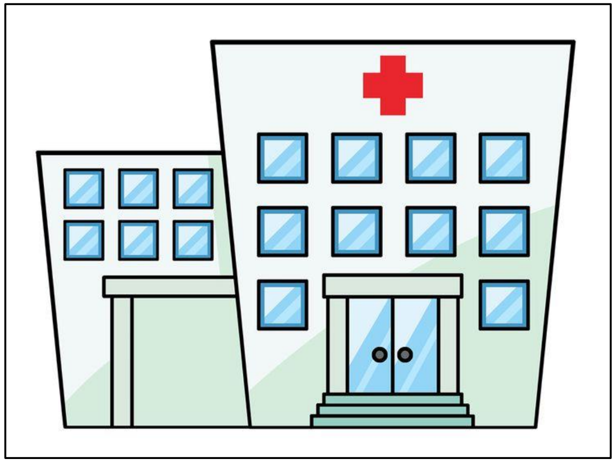


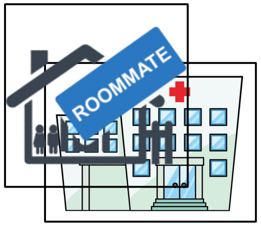


Roommate, Hospitals/Res.

Hospitals/Residents. A set of hospitals *H*, and a set of residents *R*. Every agent has a ranked set of acceptable partners. Every hospital has a lower quota and an upper quota.

Goal. Does there exist a stable matching?

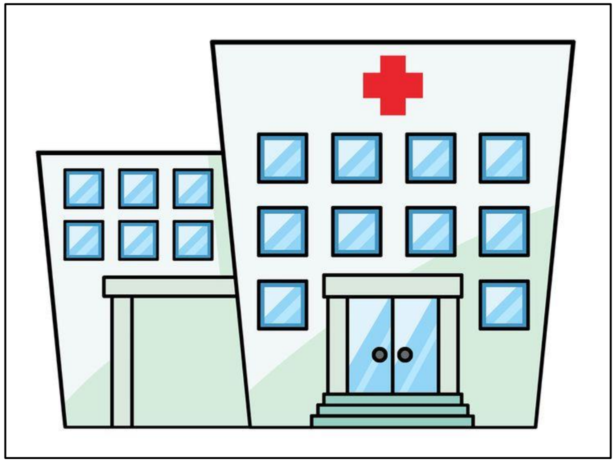


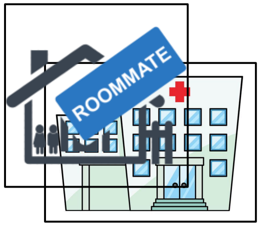


Roommate, Hospitals/Res.

Hospitals/Residents. A set of hospitals *H*, and a set of residents *R*. Every agent has a ranked set of acceptable partners. Every hospital has a lower quota and an upper quota.

No ties: Polynomial-time.
Ties: NP-hard.





Roommate, Hospitals/Res.

Proposition. Hospitals/Residents with Couples without lower quotas is W[1]-hard parameterized by the number of couples. [Marx and Schlotter, ‘11]

In addition, they studied strict and permissive local search versions of this problem.

Proposition. max-Hospitals/Residents is FPT w.r.t. number of ``agent types’’. [Meeks and Rastegari, 2017]

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